

## SF: Back to theory

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I see a lot of negative postings about my ideas, and surrogate factoring is getting a lot of bashing, but hey, it's just an idea.

It may not be practical—ever. But it's still just an idea, and I can discuss it as just an idea, having long ago backed away from calling it a solution to the factoring problem.

Now some posters seem to be obsessed with political posting meant to drive others away from my idea. I say, look at what they're doing as just that—political postings.

Now to the theory.

Basically with the latest surrogate factoring I've been analyzing the quadratics:

$$yx^2 + Ax - M^2 = 0$$

and

$$yz^2 + Az - j^2 = 0$$

$$\text{where } T = M^2 - j^2$$

and my algorithms focus on  $y$ , for several reasons, not the least of which it \*seems\* to only need the factorization of  $T$ , while other algorithms, which I'll get to later, require that both  $j$  and  $T$  be factored, while doing far worse than algorithms that just use factoring  $T$ .

Now an easy thing to do is just subtract the first equation from the second:

$$y(x^2 - z^2) + A(x-z) - M^2 + j^2 = 0,$$

and, of course,  $M^2 - j^2 = T$ , so

$$y(x^2 - z^2) + A(x-z) - T = 0,$$

$$y(x^2 - z^2) = T - A(x-z)$$

which gives that

$$y = (T + A(z-x))/(x^2 - z^2)$$

so I can kind of look at y, to the extent that you can tell anything from that equation.

One thing that is clear is that the denominator of y for rational y's must be a perfect square, which you can see easily enough by solving for x with the first quadratic to get

$$x = (-A \pm \sqrt{A^2 + 4My})/2y$$

and that is visible by inspection.

Notice also I can go back to my solution for y, and divide both sides by  $A^2$  to get

$$y/A^2 = (T + A(z-x))/((Ax)^2 - (Az)^2)$$

where the equations again show a lack of dependency on the value of A, as it can be wrapped up into other expressions, which is a feature shown again when y is solved out, so that you have

$$Ax = Az(-Az \pm \sqrt{(Az - 2M^2)^2 - 4TM^2})/(2j^2 - 2Az)$$

and

$$Az = Ax(-Ax \pm \sqrt{(Ax - 2j^2)^2 + 4Tj^2})/(2M^2 - 2Ax)$$

which also shows that an integer Az must exist, with a rational Ax, that factors M.

The problem has been figuring out how to find that integer Az or rational Ax, as it's \*easy\* to get integer Ax, but from postings on the sci.crypt newsgroup from people who claim to have tried it, it doesn't seem to factor all that often.

But you can see two things, that a solution must exist, and that A can be wrapped up into x and z, such that its value need not be determined.

Looking again at

$$y/A^2 = (T + (Az - Ax))/((Ax)^2 - (Az)^2)$$

you can see an assumption I'm making in my algorithms, where I basically assume that after common factors between

$$((Ax)^2 - (Az)^2)$$

and

$$(T + (Az - Ax))$$

are divided off, what remains only shares prime factors with T.

Now that is the assumption I make in my algorithms, and it comes from a solution I have for y, where I've related it to its own factor, and the equation I solve to get that solution is

$$f_1^2 s_1^4 - (A^2 + 4j^2 y + 2Ty)s_1^2 + f_2^2 y^2 = 0$$

where  $f_1 f_2 = T$ , and  $s_1 s_2 = y$  (the equation for  $s_2$  differs only by indices),

so I can look at how y is related to its own factor, and notice that the far left term has  $f_1^2$  as a coefficient.

With no other factors visible, it seems reasonable to suppose that the denominator of y has prime factors of T only, but algorithms based on that idea do not always factor, and I don't know why.

Now if you wish to derive that equation relating y to its own factor, it's not hard to do, but I'd just as soon refer you to the paper that steps through it, rather than make this post overly complicated.

One thing worth mentioning is that explicit equations relating x and y to factors of T and j are easily derived:

$$y = A^2(f_1 - b_1)(f_2 - b_2)/(b_1 f_2 + b_2 f_1 - 2b_1 b_2)^2$$

where  $b_1 b_2 = -j^2$  and  $f_1 f_2 = T$ ,

and

$$x = (b_1 f_2 + b_2 f_1 - 2b_1 b_2)/A.$$

With all the detail available, it's of some interest that the complete solution to the prime factors that make up the denominator of y seems to be a hard problem, as with that solution, surrogate factoring can be made to work perfectly.

I still can't see why T doesn't provide all of those factors, given the equation relating y to its own factor, but there must be some reason, or the algorithms I've tried so far would work.

All of this may be of "pure math" interest only, if these equations can't be turned into practical algorithms.

James Harris